Surface area estimation of level sets: Interaction between anisotropic directions and tessellations

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PROGRAMME DE RECHERCHE
RISQUES

Rain patterns: a real-data motivation

- Study of geometric patterns in rainfall: AghaKouchak et al. (2011)

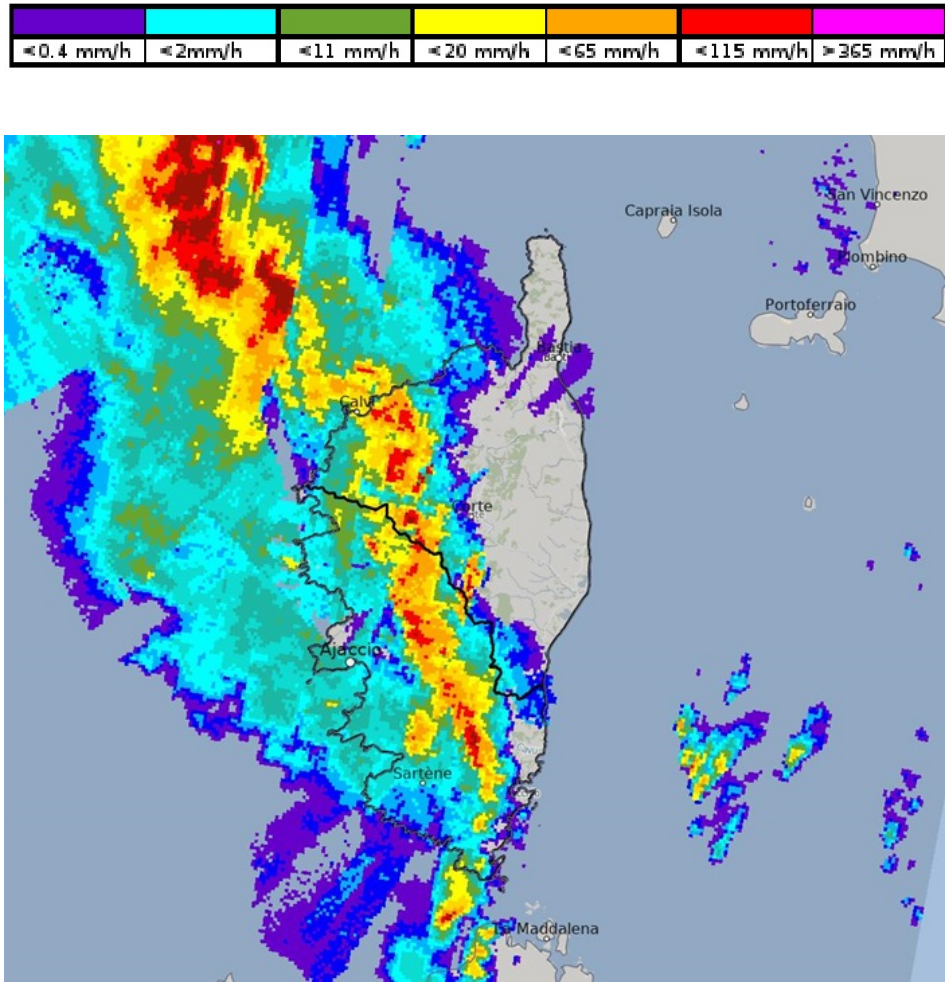

Figure: Radar picture of storm on August 28, 2025, 5:30 PM UTC - Credit : Météo-France

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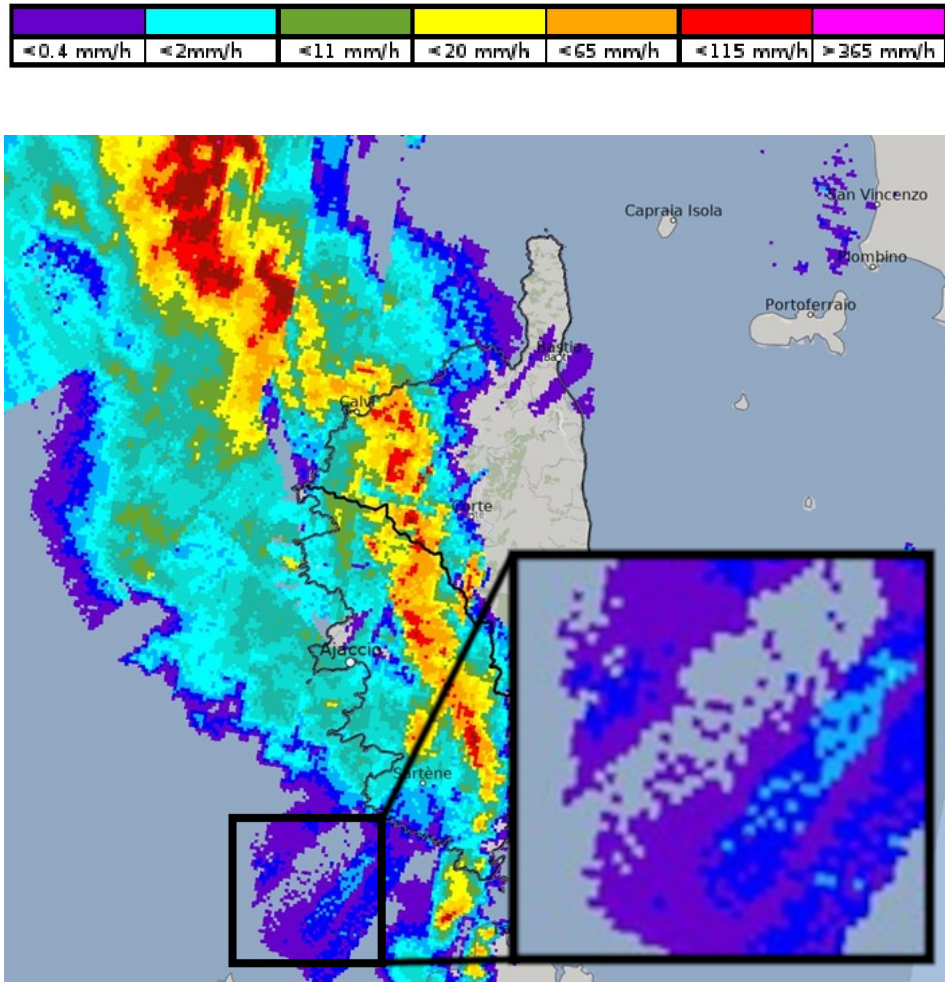


Figure: Radar picture of storm on August 28, 2025, 5:30 PM UTC - Credit : Météo-France

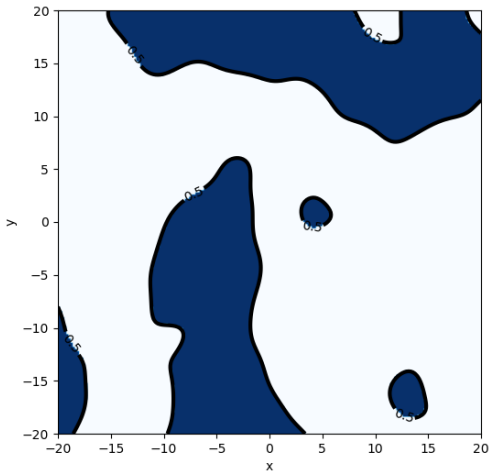
Main object

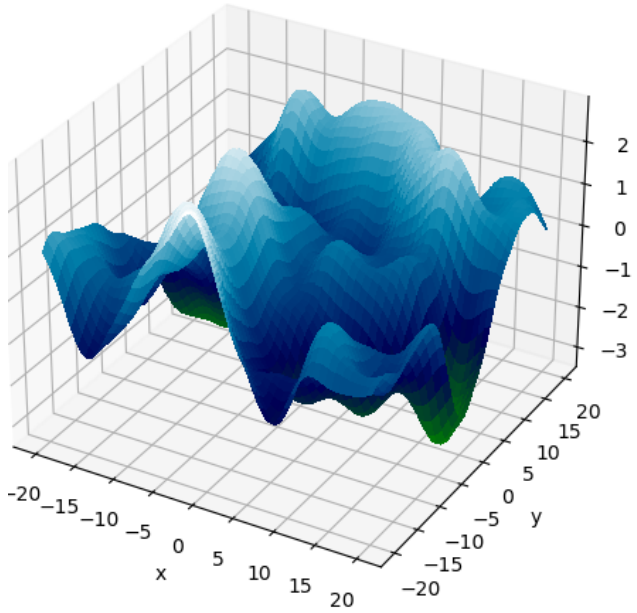
Definition: *Random field*

A random field is a measurable map $X : \Omega \times \mathbb{R}^d \to \mathbb{R}$, $d \geq 2$.

Main hypotheses:

- X is *smooth*.
- X is *stationary*.



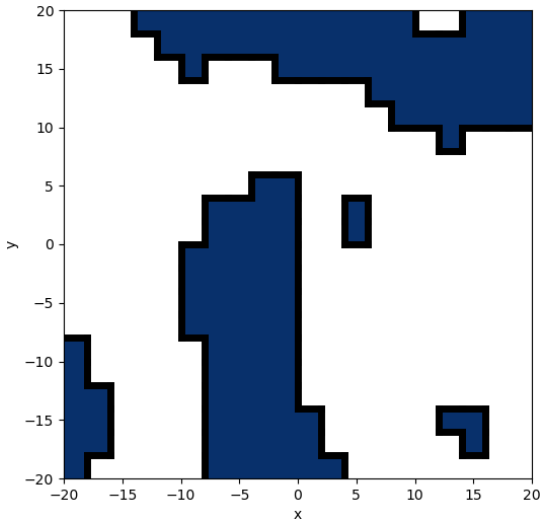


Definition: *Excursion set*

$$E_X(u) := \left\{ t \in \mathbb{R}^d ; X(t) \geq u \right\}.$$

Discretized Excursion Set

- Excursion set is observed:
  - over a compact ***observation window*** $T \subset \mathbb{R}^d$;
  - numerically approximated on a ***tessellation*** $\mathcal{H}$ of $\mathbb{R}^d$.



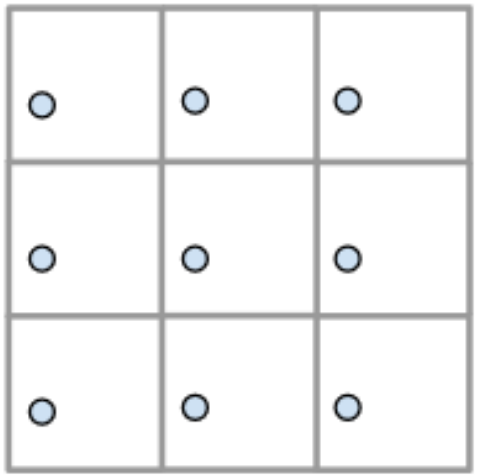
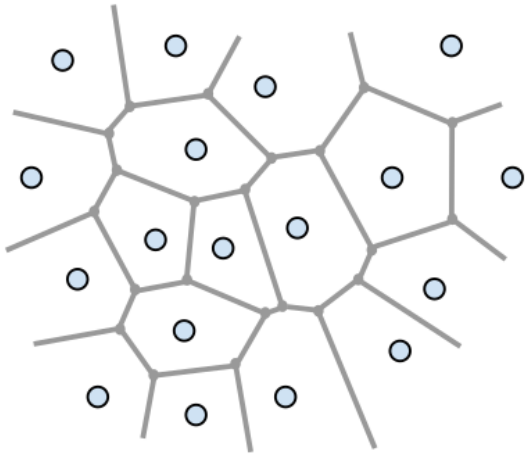


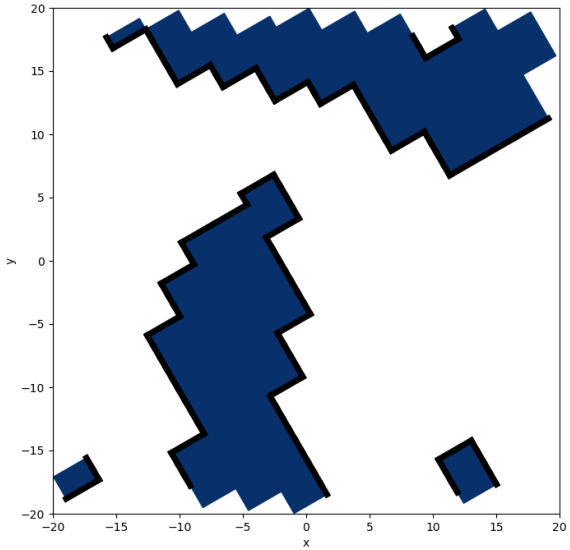
Figure: $E_X^T(\mathcal{H}^\bullet)$ for $T = [-20, 20]^2$, in square tessellation (left), with associated point-referenced tessellation (right).

Honeycomb polytopic tessellations

Definition:

$\mathcal{H}^\bullet = \{(P, P^\bullet), P \in \mathcal{H}\}$, with $P^\bullet \in P$, P polytopes tiling $\mathbb{R}^d$. It is **honeycomb** if $(P^\bullet - Q^\bullet) \perp P \cap Q$ for adjacents P, Q.





Definition: *Discretized excursion set*

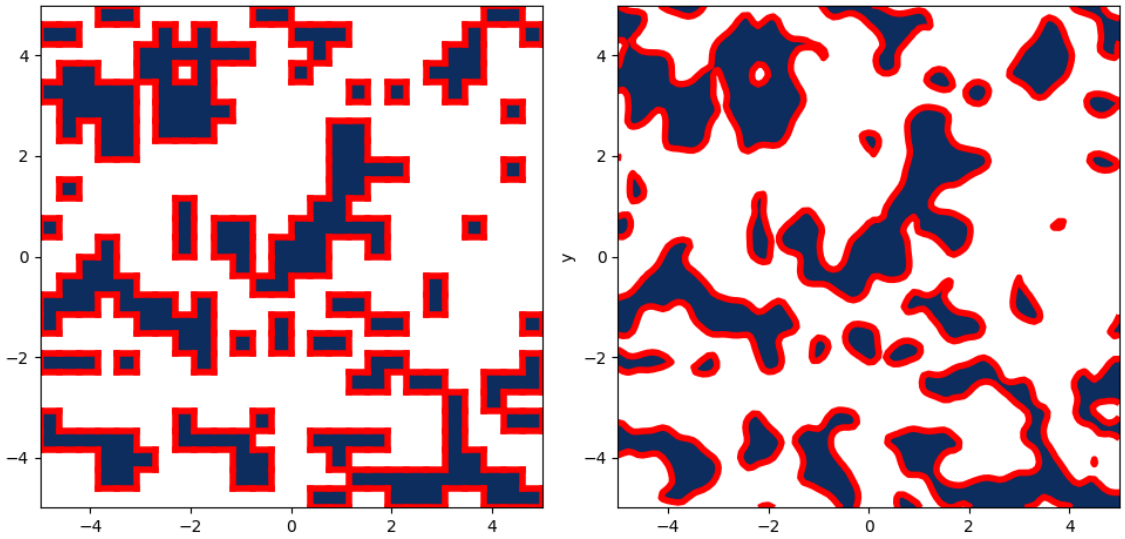
$E_X^T(\mathcal{H}^\bullet) = \bigcup_{P \subset T; X(P^\bullet) \geq u} P.$

Surface Area: $d-1$ Hausdorff measure

Surface Area:

- $C_{d-1}^{T}(X,u) := \mathbb{E}\left[\sigma_{d-1}\left(T \cap \partial E_X(u)\right)\right]$,
- **Estimator:** $\widehat{C}_{d-1}^{T}(\mathcal{H}^{\bullet}, X, u) = \sigma_{d-1}\left(T \cap \partial E_X(\mathcal{H}^{\bullet}, u)\right)$ is the length of pixelized red line.

- σ_{d-1}: $d-1$ *Hausdorff measure* in $\mathbb{R}^d$. Definition



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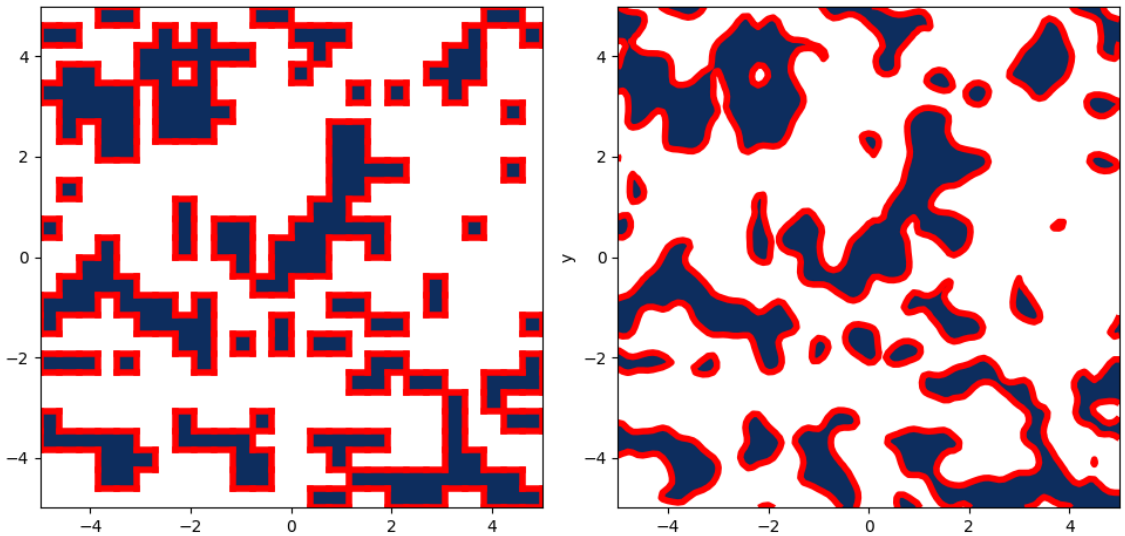
- $\mathbb{E}\left[\widehat{C}_{d-1}^{T}(\delta\mathcal{H}^{\bullet}, X, u)\right] \xrightarrow[\delta\to 0]{} C_{d-1}^{T}(X,u)$ **???**
- Biermé and Desolneux (2021): hexagonal and square case.



Connection to literature

Theorem: [*Cotsakis et al. (2024)*]

Suppose X is stationary, **isotropic**, and satisfies additional regularity assumption $(\mathcal{A}0)$, and that $\mathcal{H}^\bullet$ is **honeycomb tesselation**, then, if $C_{d-1}^T(X,u) := \mathbb{E}\left[\sigma_{d-1}\left(T \cap \partial E_X(u)\right)\right]$:

$$\mathbb{E}\left[\widehat{C}_{d-1}^T\left(\delta\mathcal{H}^\bullet, X, u\right)\right] \underset{\delta\to 0}{\longrightarrow} \frac{2d}{\beta_d} C_{d-1}^T(X,u),$$

where $\beta_d = 2\sqrt{\pi}\Gamma\left((d+1)/2\right)/\Gamma(d/2)$ is a dimensional constant.

Idea of Proof: Details

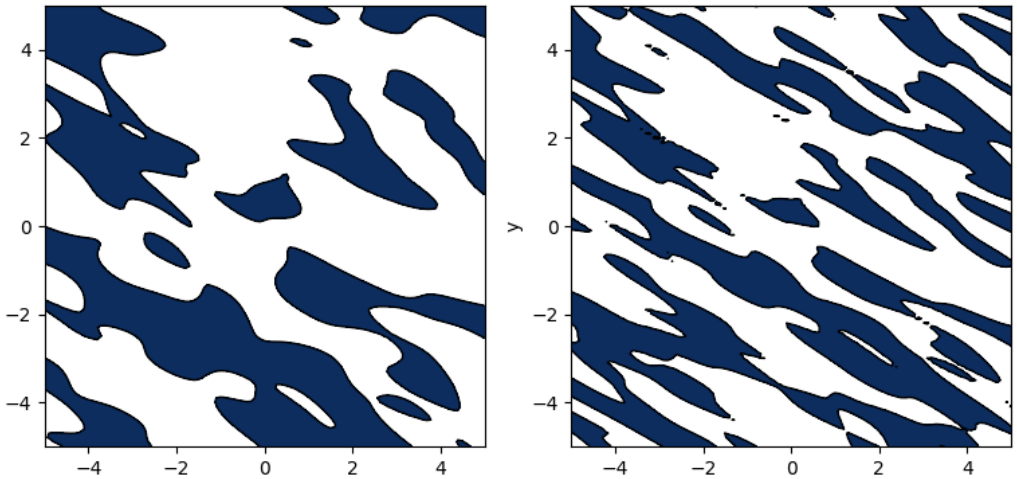
Questions:

- **Anisotropic** setting?
- **Non-honeycomb** tessellations?

Isotropy

Definition: *Isotropy*

X is isotropic $\iff \forall \rho \in SO(d), X \circ \rho \overset{(d)}{=} X.$



- **Proposition:** X isotropic $\iff \forall \rho, \rho((E_X(u))_{u\in\mathbb{R}}) \overset{(d)}{=} (E_X(u))_{u\in\mathbb{R}}.$

Anisotropy of deformed fields

Definition: *Deformed field*

The field $X_\Phi = \left\{X(\Phi(t)), t \in \mathbb{R}^d\right\}$, for $\Phi : \mathbb{R}^d \to \mathbb{R}^d$ **linear bijective** and X **isotropic** (Estrade and Fournier (2024)).

- **Properties:** Suppose X is isotropic and satisfy $(\mathcal{A}0)$, then:
  1) X_Φ isotropic $\iff$ $\Phi = \lambda\rho, \rho \in O(d)$;
  2) $E_{X_\Phi}(u) = \Phi^{-1}(E_X(u))$ a.s.;
  3) $C^T_{d-1}(X_\Phi, u) = \gamma_\Phi C^T_{d-1}(X, u)$.
- $\gamma_\Phi = \dfrac{\sigma_{d-1}(\Phi^{-1}(\mathbb{S}^{d-1}))}{d\,|\Phi^{-1}(\mathbb{B}^d)|}$ is an *isoperimetric ratio* for $\Phi^{-1}(\mathbb{B}^d)$.

Main Assumption ($\mathcal{A}0$)

Kac-Rice formula:

$$C_{d-1}^{T}(X,u) = \mathbb{E}\left[\sigma_{d-1}\left(T \cap \partial E_X(u)\right)\right] = |T|\, f_{X(0)}(u)\mathbb{E}\left[\|\nabla X(0)\|_2 \,|X(0) = u\right].$$

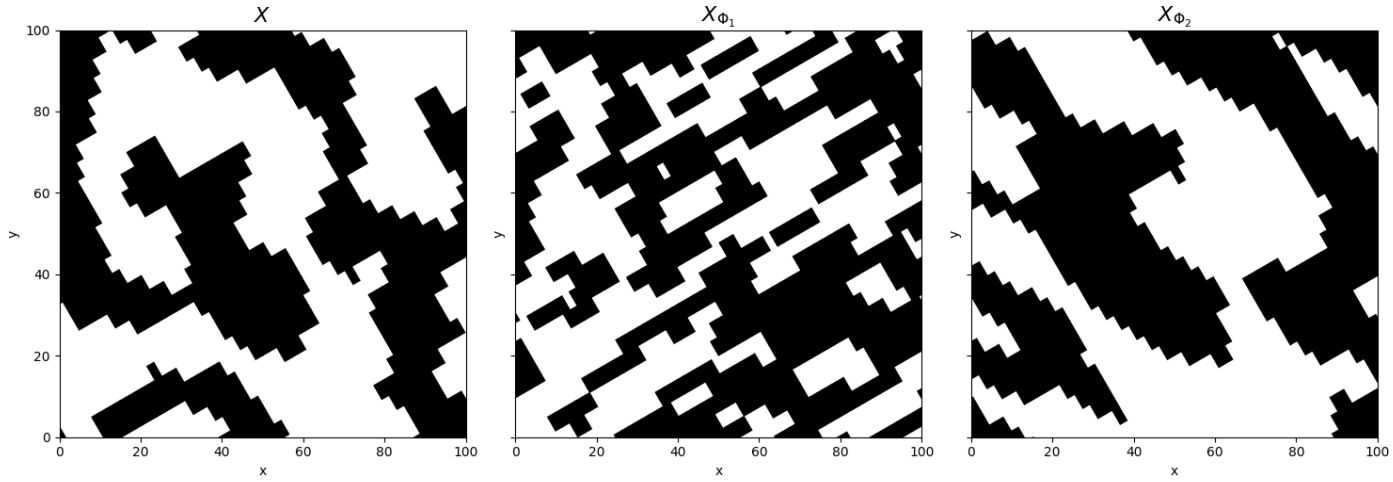
Assumption ($\mathcal{A}0$): [*Armentano et al. (2025)*]

- X is **smooth**, **stationary**;
- $(X(0), \nabla X(0))$ admits a **bounded density** $f_{X(0),\nabla X(0)}$ with integrability condition;
- $u \mapsto \mathbb{E}\left[\|\nabla X(0)\| \,|X(0) = u\right]$ is **continuous**.

Then, $E_X(u)$ satisfies: Stationary ✓, Smooth ✓, $C_{d-1}^{T}(X,u) < \infty$ ✓.

Example: Gaussian-type fields: $t \mapsto F\left(G_1(t), \cdots G_k(t)\right)$, where $F : \mathbb{R}^k \to \mathbb{R}$ regular and $G_1, \cdots, G_k$ Gaussian.

Toy case: Rotated rectangles



Theorem:

Suppose X is **isotropic**, satisfies $(\mathcal{A}0)$, and that Δ is a rectangle tessellation. If $\rho_0 \in SO(d)$, and $\Phi : \mathbb{R}^d \to \mathbb{R}^d$ is linear bijective, then

$$\mathbb{E}\left[\widehat{C}_{d-1}^{T}\left(\delta\rho_0(\Delta^{\bullet}), X_{\Phi}, u\right)\right] \underset{\delta\to 0}{\longrightarrow} \frac{2d}{\beta_d}\frac{1}{\gamma_{\Phi}}\frac{\sum_{1\leq i\leq d}\|\Phi(\rho_0(e_i))\|_2}{d} C_{d-1}^{T}(X_{\Phi}, u).$$

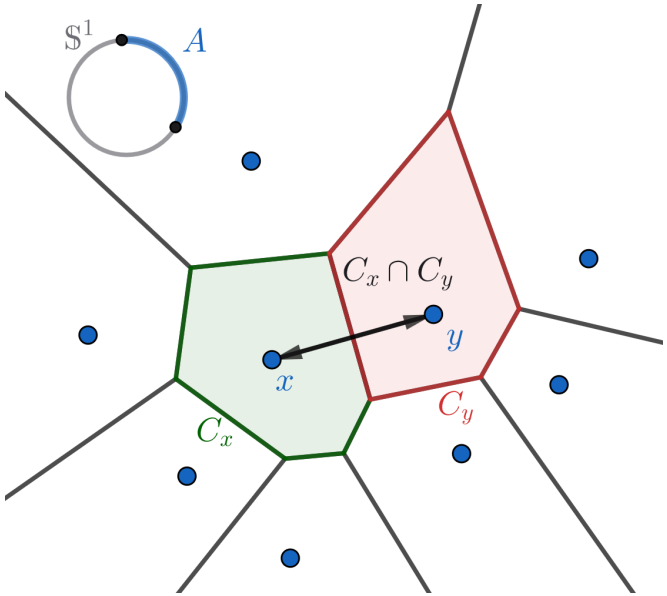
- $\rho_0(e_i)$: geometric features of tessellation.

Main object

Definition: *Directional measures* Details

Fixed T observation window. Measure $\mu_{\mathcal{H}}(T, \cdot)$ on $\mathbb{S}^{d-1}$:

$$\mu_{\mathcal{H}}(T, A) = \sum_{\substack{x,y \in \mathcal{H} \\ C_x, C_y \subset T}} \sigma_{d-1}(C_x \cap C_y) \, \|x - y\|_2 \, \delta_{\frac{x-y}{\|x-y\|}}(A), \qquad A \subset \mathbb{S}^{d-1}.$$



- **Square case, $d = 2$:**

$$\mu_{\Delta}(T, A) = \sum_{\substack{x,y \in \mathcal{H} \\ C_x, C_y \subset T}} a^2 \delta_{\frac{x-y}{\|x-y\|}}(A)$$

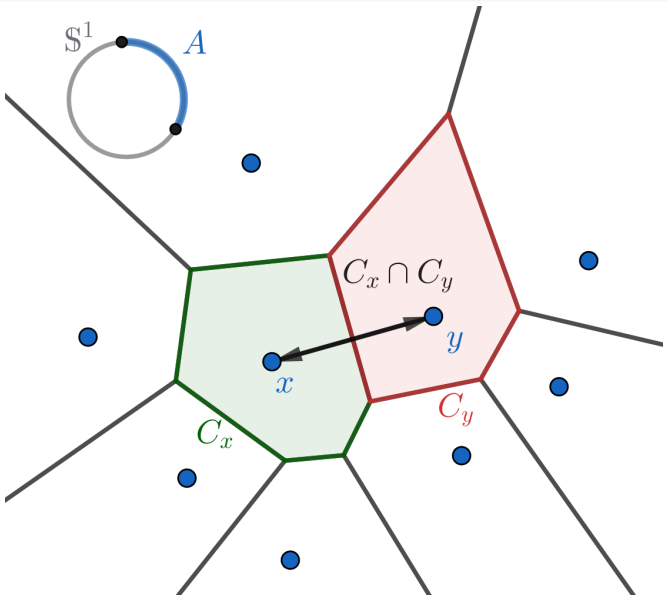
$$= a^2 N \left(\delta_{\pm e_1} + \delta_{\pm e_2}\right)(A)$$

Main object

Definition: *Directional measures* Details

Fixed T observation window. Measure $\mu_{\mathcal{H}}(T, \cdot)$ on $\mathbb{S}^{d-1}$:

$$\mu_{\mathcal{H}}(T, A) = \sum_{\substack{x,y \in \mathcal{H} \\ C_x, C_y \subset T}} \sigma_{d-1}(C_x \cap C_y) \, \|x - y\|_2 \, \delta_{\frac{x-y}{\|x-y\|}}(A), \qquad A \subset \mathbb{S}^{d-1}.$$



Assumption $(\mathcal{A}1)$:

- There exists $\mu^\star_{\mathcal{H}}$ such that for any T, $\dfrac{\mu_{\delta\mathcal{H}}(T, \cdot)}{|T|} \xrightarrow[\delta \to 0]{(w)} \mu^\star_{\mathcal{H}}$;
- $\sup\limits_{\substack{x \sim y \in \delta\mathcal{H} \\ C_x, C_y \subset T}} \{\|x - y\|_2\} \xrightarrow[\delta \to 0]{} 0.$

Main result

Theorem:

Suppose X is **isotropic**, satisfies $(\mathcal{A}0)$, and that $\mathcal{H}^\bullet$ is a tessellation satisfying Assumption $(\mathcal{A}1)$. If $\Phi : \mathbb{R}^d \to \mathbb{R}^d$ is linear bijective, then

$$\mathbb{E}\left[\widehat{C}_{d-1}^T(\delta\mathcal{H}^\bullet, X_\Phi, u)\right] \underset{\delta\to 0}{\longrightarrow} \frac{2d}{\beta_d}\frac{1}{\gamma_\Phi}\frac{1}{d}\int_{\mathbb{S}^{d-1}} \|\Phi(s)\|_2\, \mu_{\mathcal{H}}^\star(ds)\, C_{d-1}^T(X_\Phi, u).$$

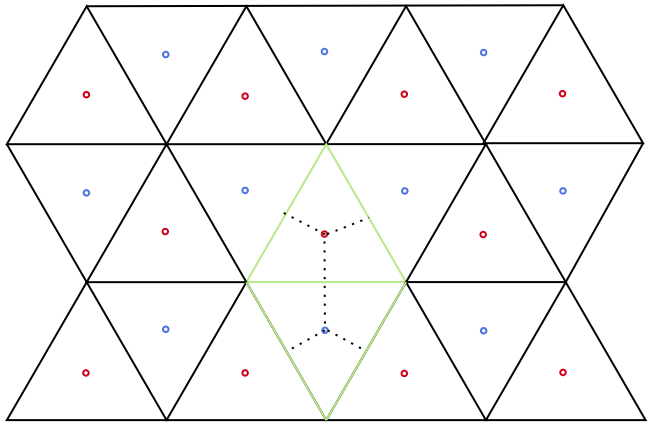
Question:

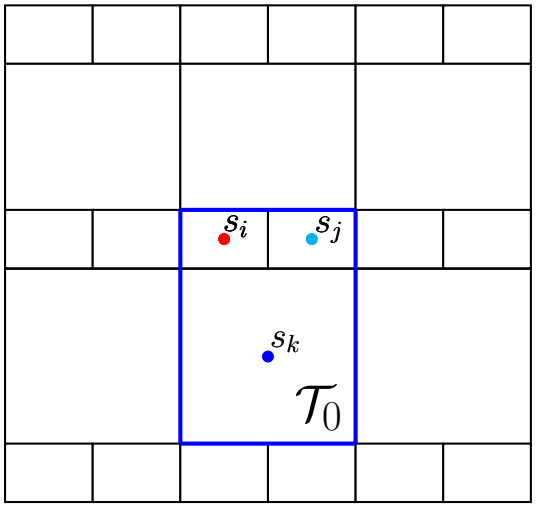
- Tessellations satisfying $(\mathcal{A}1)$?
- Explicit form of $\mu_{\mathcal{H}}^\star$?

Periodic tiles

Periodic Tiling:

There exists a lattice $\mathbb{L} \subset \mathbb{R}^d$ and a finite proto-tile $\mathcal{T}_0^\bullet$ such that $\mathcal{P}^\bullet = \cup_{\ell \in \mathbb{L}} (\ell + \mathcal{T}_0^\bullet)$.





$\mu_{\mathcal{P}}^\star$:

$$\mu_{\mathcal{P}}^\star = \sum_{x \sim y} \frac{\|(y-x) \cap \mathcal{T}_0\| \, \sigma_{d-1}(C_x \cap C_y)}{|\mathcal{T}_0|} \delta_{\vec{xy}}$$

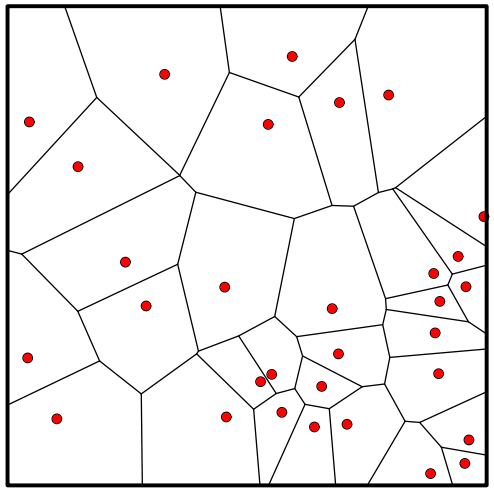
- **Rotated rectangle $\rho_0(\Delta)$ case:**

$$\mu_{\rho_0(\Delta)}^\star = \sum_{1 \le i \le d} \delta_{\rho_0(e_i)} + \delta_{-\rho_0(e_i)}.$$

Stationary-ergodic Voronoï tiles

Voronoï Tiling:

If ξ is a point process on $\mathbb{R}^d$, $\mathcal{V}^\bullet(\xi) = \left\{\left(\left\{y \in \mathbb{R}^d; d(y,x) = d(y,\xi)\right\}, x\right), x \in \xi\right\}$.



Suppose that ξ satisfy:

- ξ is **stationary**;
- ξ is **ergodic**.

For instance, ξ is an homogeneous Poisson Point Process.

$\mu^\star_{\mathcal{V}(\xi)}$:

If ξ is additionally **isotropic**:

$$\mu^\star_{\mathcal{V}(\xi)}(M) = 2d\frac{\sigma_{d-1}(M)}{\sigma_{d-1}(\mathbb{S}^{d-1})}.$$

Sketch of Proof 1/2

Sketch of Proof: For fixed $f : \mathbb{S}^{d-1} \to \mathbb{R}$ measurable bounded,

$$\mu_{\mathcal{V}(\xi)}(T, f) = \sum_{i \neq j:\ C_i, C_j \in \mathcal{V}(\xi)} \sigma_{d-1}(C_i \cap C_j)\sigma_1(\ell_{ij}) f(\vec{\theta}_{ij}) \mathbb{1}_{C_i, C_j \subset T}.$$

Assumption ($\mathcal{A}1$):

- There exists $\mu^\star_{\mathcal{V}(\xi)}$ such that $\dfrac{\mu_{\delta\mathcal{V}(\xi)}(T, \cdot)}{|T|} \xrightarrow[\delta \to 0]{(w)} \mu^\star_{\mathcal{V}(\xi)}$;
- $\sup\limits_{\substack{x \sim y \in \delta\xi \\ C_x, C_y \subset T}} \{\|x - y\|_2\} \xrightarrow[\delta \to 0]{} 0.$

1) *Approximation of $\mu_{\mathcal{V}(\xi)}(\cdot, f)$ by a measure $\Pi_{\mathcal{V}(\xi)}(f, \cdot)$:*

$$\Pi_{\mathcal{V}(\xi)}(f, \cdot) := \sum_{s_i \in \xi \cap T} \sum_{s_j \sim s_i} \sigma_{d-1}(C_i \cap C_j)\sigma_1(\ell_{ij}) f(\vec{\theta}_{ij}).$$

ξ stationary ergodic $\implies$ $\Pi_{\mathcal{V}(\xi)}(f, \cdot)$ stationary ergodic.

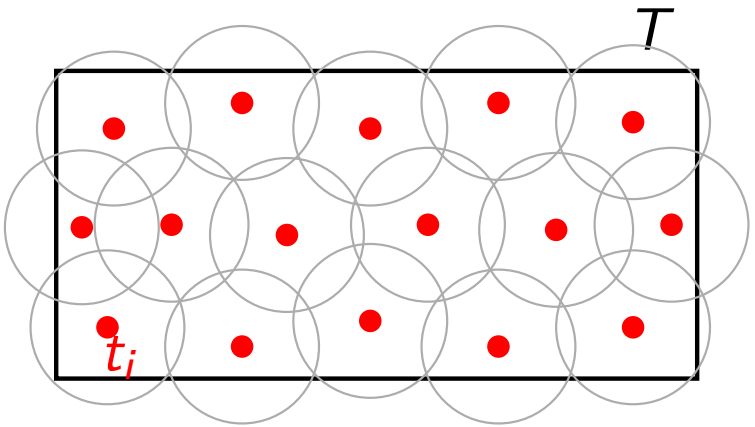
Sketch of Proof 2/2

2) *Ergodic Theorem to identify* $\Pi^{\star}_{\mathcal{V}(\xi)}(f,\cdot)$*:* By [*Nguyen and Zessin (1979)*]

$$\frac{\Pi_{\delta\mathcal{V}(\xi)}(f,T)}{|T|}=\frac{\Pi_{\mathcal{V}(\xi)}(f,\delta^{-1}T)}{|\delta^{-1}T|}\xrightarrow[\delta\to 0]{a.s.}\Pi^{\star}_{\mathcal{V}(\xi)}(f).$$

3) $\sup_{\substack{x\sim y\in\delta\xi\\ C_x,C_y\subset T}}\{\|x-y\|_2\}\xrightarrow[\delta\to 0]{a.s.}0.$

Idea: $T\subset\bigcup B(t_i,\varepsilon)$ with ultimately $(\delta\xi)(B(t_i,\varepsilon))>0,\forall i$ a.s. by Ergodic Theorem.



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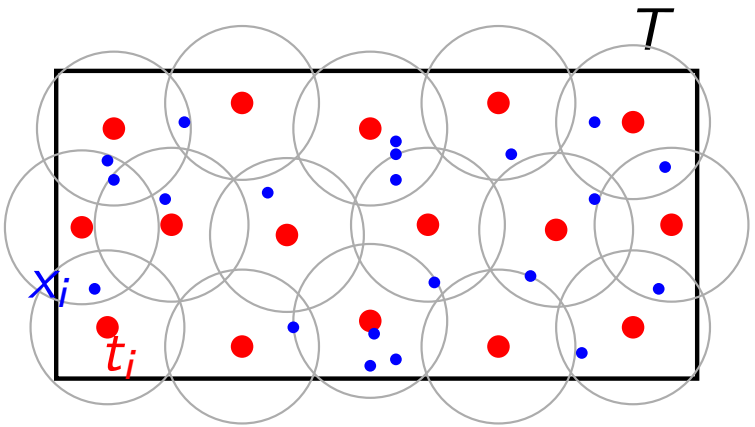

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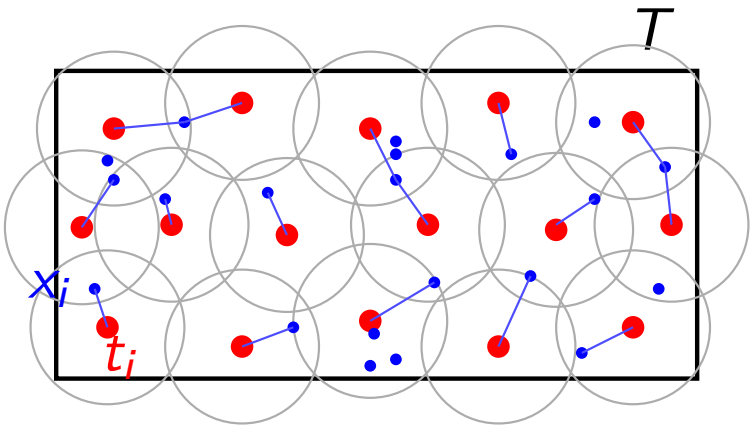

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4) *Edges effect:*

$$\eta_{\delta\mathcal{V}(\xi)}(f,T):=\frac{\Pi_{\delta\mathcal{V}(\xi)}(f,T)-\mu_{\delta\mathcal{V}(\xi)}(T,f)}{|T|}\xrightarrow[\delta\to 0]{a.s.}0.$$

Idea: $\eta_{\delta\mathcal{V}(\xi)}(f,T)\simeq\frac{1}{|T|}$terms ε-closed to ∂T

$$\leq\varepsilon+\alpha(f,T)\mathbb{1}_{\text{diam}(\delta C_x)\geq\varepsilon\text{ or }\text{diam}(\delta C_y)\geq\varepsilon}\xrightarrow[\delta\to 0]{a.s.}0.$$

Conclusion

- Non-universal bias: depends on the action of anisotropy on the tessellation;
- Characterized bias for large classes of tessellations: periodic and stationary ergodic Voronoï;
- Results are in:

  Di Bernardino, Duval, and Ledieu, *Surface area estimation of level sets: Interaction between anisotropic directions and tessellations* (2026)

Amir AghaKouchak, Nasrin Nasrollahi, Jingjing Li, Bisher Imam, and Soroosh Sorooshian. Geometrical characterization of precipitation patterns. *Journal of Hydrometeorology*, 12(2):274–285, 2011.

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Anne Estrade and Julie Fournier. Gaussian Fields through Geometrical Properties. *hal-04481440*, 2024.

Xuan Xanh Nguyen and Hans Zessin. Ergodic theorems for spatial processes. *Zeitschrift für Wahrscheinlichkeitstheorie und Verwandte Gebiete*, 48(2): 133–158, 1979.

Idea of Proof 1/2

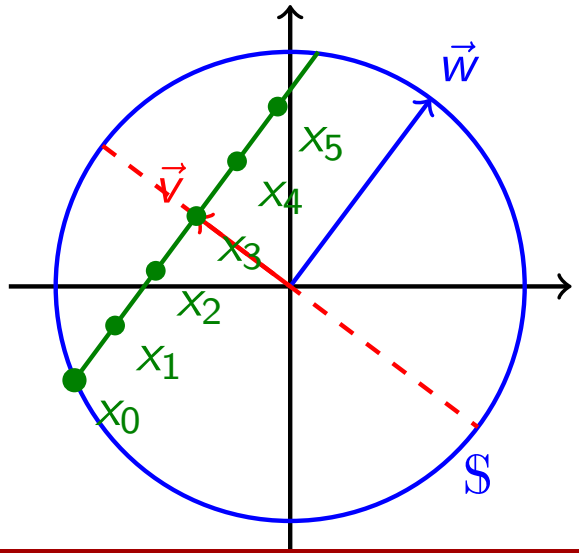
Back to slide

By discretization method, we approximate ***Crofton formula*** for $T = \mathbb{B}^d$:

$$\mathbb{P}(X(0) \leq u < X(\delta w)) \int_{\mathbb{B}^d \cap w^\perp} n_\delta(v) dv = \int_{\mathbb{B}^b \cap w^\perp} \mathbb{E}\left[\sum_{i=0}^{n_\delta(v)-1} \mathbb{1}_{X(x_j) \leq u < X(x_{j+1})} \right] dv$$

$$\xrightarrow[\delta \to 0]{} \int_{w^\perp} \mathbb{E}\left[\#\left\{ t \in \ell_{w,v}; X(t) = u \right\} \right] dv$$

$$= \frac{1}{\beta_d} \frac{C_{d-1}^T(X, u)}{|T|}.$$



Idea of Proof 2/2

$$\mathbb{E}\left[\widehat{C}_{d-1}^{T}\left(\delta\mathcal{H}^{\bullet}, X, u\right)\right] = \sum_{\substack{x,y\in\delta\mathcal{H}^{\bullet}\\ C_x, C_y\subset T}} \sigma_{d-1}(C_x\cap C_y)\mathbb{E}\left[\mathbb{1}_{X(x)\leq u<X(y)}\right]$$

$$= \sum_{\substack{x,y\in\delta\mathcal{H}^{\bullet}\\ C_x, C_y\subset T}} \sigma_{d-1}(C_x\cap C_y)\mathbb{P}\Big(X(0)\leq u < X(x-y)\Big).$$

$$\underset{\delta\to 0}{\simeq} \frac{1}{\beta_d}\frac{C_{d-1}^{T}(X,u)}{|T|}\sum_{\substack{x,y\in\delta\mathcal{H}^{\bullet}\\ C_x, C_y\subset T}} \sigma_{d-1}(C_x\cap C_y)\|x-y\|_2$$

$$\underset{\delta\to 0}{\longrightarrow} \frac{2d}{\beta_d} C_{d-1}^{T}(X,u)$$

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Hausdorff measure

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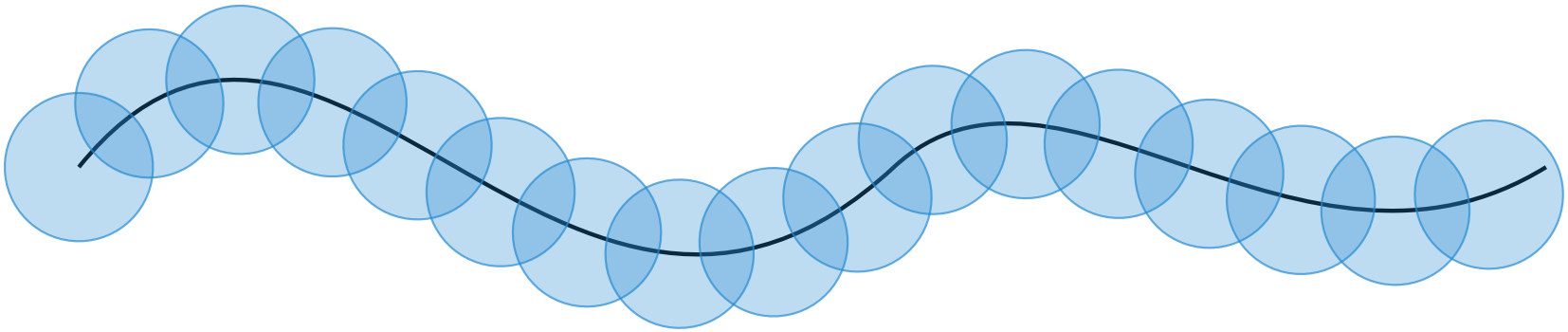
- *Hausdorff measure*: $\sigma_{d-1}(A) = \dfrac{\pi^{(d-1)/2}}{2^{d-1}\Gamma(1+(d-1)/2)} \lim_{\delta \to 0} \sigma^{\delta}_{d-1}(B)$,
- $\sigma^{\delta}_{d-1}(B) = \inf \left\{ \sum_{i \geq 1} \mathrm{diam}(U_i)^{d-1}, \mathrm{diam}(U_i) \leq \delta \right\}$.



Directional measure of tessellation 1/2

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$$\widehat{C}_{d-1}^{T}(\delta\mathcal{H}^{\bullet}, X_{\Phi}, u) = \sum_{\substack{x,y\in\delta\mathcal{H}^{\bullet}\\ C_x,C_y\subset T}} \sigma_{d-1}(C_x\cap C_y)\mathbb{1}_{X(x)\leq u<X(y)}.$$

$$\mathbb{E}\left[\widehat{C}_{d-1}^{T}(\delta\mathcal{H}^{\bullet}, X_{\Phi}, u)\right] = \sum_{\substack{x,y\in\delta\mathcal{H}^{\bullet}\\ C_x,C_y\subset T}} \sigma_{d-1}(C_x\cap C_y)\mathbb{P}\left(X(\Phi(x))\leq u < X(\Phi(y))\right)$$

$$= \sum_{\substack{x,y\in\delta\mathcal{H}^{\bullet}\\ C_x,C_y\subset T}} \sigma_{d-1}(C_x\cap C_y)\mathbb{P}\left(X(0)\leq u < X(\Phi(y-x))\right)$$

$$\underset{\delta\to 0}{\simeq} \sum_{\substack{x,y\in\delta\mathcal{H}^{\bullet}\\ C_x,C_y\subset T}} \sigma_{d-1}(C_x\cap C_y)\frac{\|\vec{xy}\|}{\beta_d\gamma_d(\Phi)} C_{d-1}^{T}(X_{\Phi}, u)\,\|\Phi(\vec{xy}/\,\|\vec{xy}\|)\|.$$

Directional measure of tessellation 2/2

$$\frac{1}{\beta_d \gamma_d(\Phi)} C_{d-1}^T(X_\Phi, u) \sum_{\substack{x,y \in \delta\mathcal{H}^\bullet \\ C_x, C_y \subset T}} \sigma_{d-1}(C_x \cap C_y) \, \|\vec{xy}\| \, \left\|\Phi(\theta_{\vec{xy}})\right\|$$

For T fixed, we have the measure on $\mathbb{S}^{d-1}$:

$$\mu_{\mathcal{H}}(T, A) = \sum_{\substack{x,y \in \mathcal{H} \\ C_x, C_y \subset T}} \sigma_{d-1}(C_x \cap C_y) \, \|x - y\|_2 \, \delta_{\vec{xy}/\|\vec{xy}\|}(A), \quad A \subset \mathbb{S}^{d-1},$$

so that:

$$\mathbb{E}\left[\widehat{C}_{d-1}^T(\delta\mathcal{H}^\bullet, X_\Phi, u)\right] \underset{\delta \to 0}{\simeq} \frac{1}{\beta_d \gamma_d(\Phi)} C_{d-1}^T(X_\Phi, u) \int_{\mathbb{S}^{d-1}} \|\Phi(\theta)\| \, \mu_{\mathcal{H}}(T, d\theta).$$

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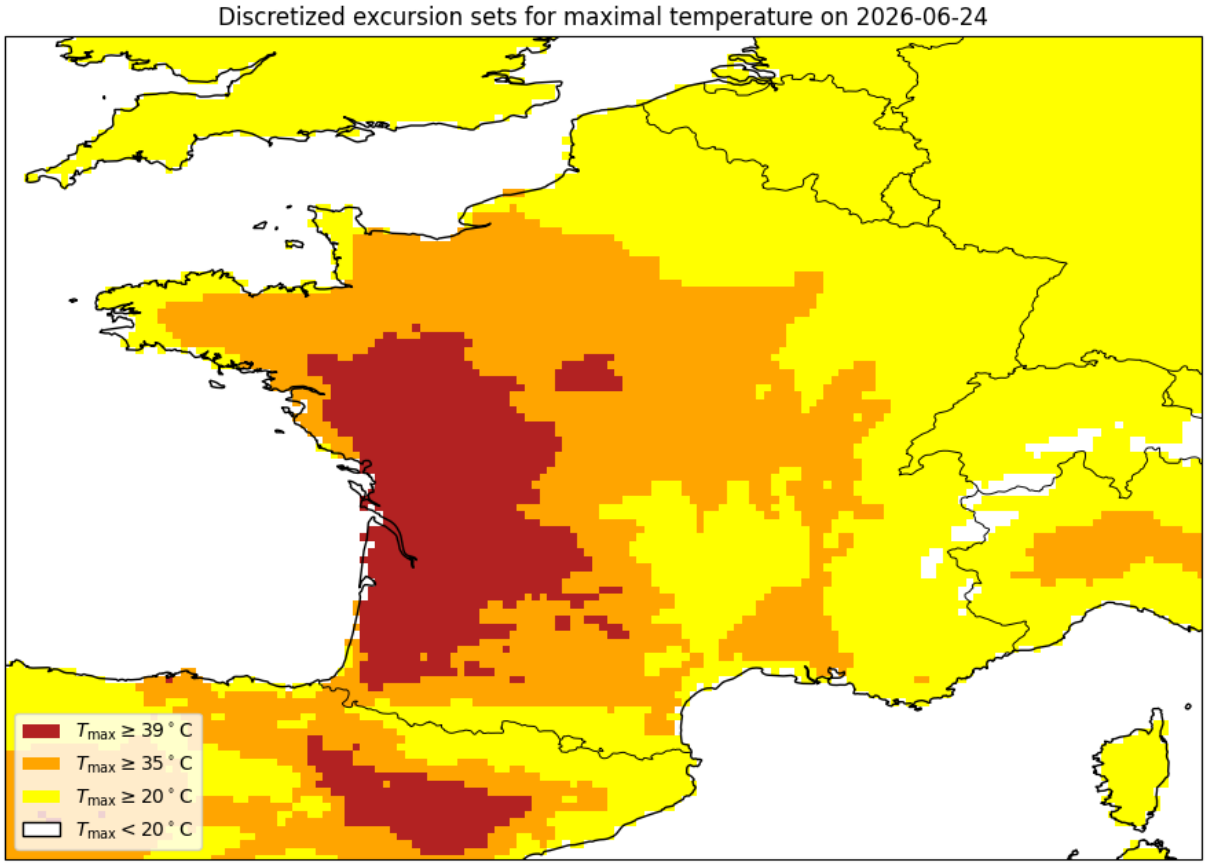
Temperature field application